Streszczenie

Advanced Optimal Traffic Control in Street Canyons

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W pracy rozwinięto zaawansowane podejście do problemów proekologicznego sterowania ruchem drogowym w miastach. Hydrodynamiczny model sterowania obejmuje wielopasmowy, jednowymiarowy, dwukierunkowy, ruch pojazdów wielu typów, wiele typów emisyjnych pojazdów, wiele typów zanieczyszczeń powietrza. Sformułowano optymalne, w sensie globalnego czasu podróży, globalnych emisji i koncentracji zanieczyszczeń, problemy proekologicznego sterowania w kanionie odosobnionym i w trzech sąsiednich kanionach zastępczych, które są reprezentatywne dla podsieci miejskiej, oraz wyciągnięto generalne wnioski co do sterowania ruchem drogowym.

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Abstract

In this paper an advanced approach to the proecological urban traffic control problems is developed. Hydrodynamical control model of the street canyon includes multilane, one-dimensional bi-directional movement of vehicles of several types, of several emission types, and of emitted pollutants. The optimal in the sense of total travel time, of pollutants' emissions, and of pollutants' concentrations, proecological control problems for an isolated street canyon, and for three adjacent substitute street canyons representative for the city subnetwork are formulated and the general management illations are deduced.

1. Introduction.

1. 1. Importance of problems that are dealt with in article.

The aim of article is minimization of road traffic influences on urban environment inhabited by humans. Such approach is natural and it is based on hydrodynamical field model [1-3]. Standard models are too general to be used in investigations of street canyons, albeit some of them might have been formally applied for street traffic control through introduction of meteorological parameters and of turbulence parameters, that artificially mimic street canyon's effects. Hereafter, we present weaknesses of these models. Standard queueing, emission, vehicular, and dispersion models: HIGHWAY-2, CALINE-3, CALINE-4, JEA and TOKYO, PREDCO, SATURN, APRAC, GZE, and PWILG, OMG, UTC-1, NETSIM, ROM, RADM, UAM, CIT, are based on simplifying assumptions [1]. Queueing models mainly disaggregate vehicular movement in four phases: cruising, deceleration, queueing, and acceleration. Automobile Exhaust Emission Modal Analysis Model (EMAM), Positive Kinetic Energy and Power Demand models assume instant vehicular velocities and accelerations in polynomial form. Traffic model of P. G. Michalopoulos, D. E. Beskos, and J. -K. Lin [4] is continuous field model. In emission models, one assumes linear emission models with constant rate of emission throughout entire road HIGHWAY-2, CALINE-3, CALINE-4, MOBILE-2, or throughout road link PREDCO [1]. Dispersion models are mainly Lagrangean probabilistic models based on assumption of existence of mobile particles (not molecules). Lagrangean models are predominantly Gaussian dispersion models, wherein assumption of concentrations in Gaussian form is made *ad hoc*. Dependence of pollutants' concentrations on wind's velocity is unilateral: concentrations depend on given wind's velocity, however wind's velocity is independent input variable. In reality, both concentrations and wind's velocity depend on each other, as well as they depend on temperature, pressure, and *vice versa*. The abovementioned models are static, not spatially distributed, simplistic, and assumed *ad hoc*. Moreover, they are not applicable for street canyons, where meteorological data (wind's velocity, temperature, pressure, density of mixture, and concentrations of constituents of mixture), as well as geometrical assumptions and their consequences (pollutants adhere on street canyon's walls, wind's velocity vanishes on walls) are important. However, there exist some models that might have been used in street canyon's modelling: SATURN, APRAC, GZE, PWILG, and OMG [1].

1. 2. Aim of article.

Aims of article are formulation and solution of proecological optimal street canyon control in street canyons by application of an advanced hydrodynamical model.

Mathematical model is built on hydrodynamical theory used for many types of air pollutants, and for many types of vehicles as well as for vehicular emissions. In model vehicles are treated as continuous one-dimensional fluids without inner structure situated on multilane bi-directional road, hence vehicular dynamics is represented by vehicular densities' fields, vehicular velocities' fields, and pollutants' emissions' fields. Equations of vehicular dynamics are balances of vehicular numbers and equations of state of vehicles [4-5]. Vehicular behaviour on signalized street junctions depends on signals' parameters and on existence of vehicular queues. Gaseous mixture is represented by

density of mass' field, pressure's field, velocity's field, temperature's field, and mixture's constituents' concentrations' fields. Dynamics of mixture is derived from laws of balances of mixture's mass, of mixture's momentum, of mixture's energy, of masses of mixture's constituents, and of equations of states of mixture and of mixture's constituents [6-15].

1. 3. Main points of article.

Vehicular movement is modelled as multilane bi-directional onelevel one-dimensional rectilinear. It is considered together with two-road traffic signalized junctions [4-5].

Gaseous mixture is composed of viscid Newtonian compressible noninteracting perfect (ideal) gases [6-15].

Model of dynamics is set of mutually interconnected nonlinear vector temporally dependent partial differential equations with nonvanishing right-hand terms (sources) together with boundary-initial problem [4-15].

Street canyon is dynamical spatially distributed control plant.

Control problems: vector of control is 5-tuple composed of two cycle's times, of two green's times, and of offset's time (time shift between starts of signalizations' cycles on two traffic junctions). The admissible set of control is defined. Six control functionals are introduced: total travel time, global pollutants' emissions, and global pollutants' concentrations for both single canyon and for three adjacent substitute street canyons of urban subnetwork. Six separate monocriterial optimal control problems are formulated.

Results: Six separate monocriterial optimization problems are solved. Analysis and verification of results are performed. Moreover, conclusions are deduced on basis of different classes of hydrodynamical and vehicular parameters' scenarios.

2. Hydrodynamical model.

2. 1. Street canyon is represented by the cuboid $\Omega = [0, a] \times [0, b] \times [0, c]$. We set Cartesian coordinate system x, y, z, at street canyon's corner. Walls are described by y=0, y=b, and road surface by z=0. Remaining three canyon's walls x=0, x=a, z=c, are composed of solely air. Walls do not have holes and vegetation throughout street is absent. Rectilinear parallel road lanes are situated on bottom of canyon. At entrance and exit of canyon there are two coordinated signalized junctions.

2. 2. Vehicles of VT emission types are modelled as fluids. Traffic movement is bi-directional.

2. 3. Gaseous mixture: The considered mixture of gases consists of $N_E - 1 + N_A = N$ gases. The first $(N_E - 1) = 3$ gases are the exhaust gases emitted by vehicle engines during combustion (CO, CH, NO_x, we neglect the presence of SO_2). The remaining $N_A = 9$ gases are the components of air

$(O_2, N_2, Ar, CO_2, Ne, He, Kr, Xe, H_2$, we neglect the presence of H_2O, O_3). The walls of the canyon and the surface of the road are impervious for all gases of the mixture. The remaining three surfaces of the cuboid are pervious for external fluxes of exhaust gases and for air components. The internal sources of air components are not present with the exception of oxygen, *i.e.*, N_E th component of the gaseous mixture. There are internal mobile sources of exhaust gases (passenger cars, and trucks, with many types of engines: diesel or petrol, and with mixed ages of engines). During the combustion the vehicular engine consumes oxygen, therefore with each internal mobile source of exhaust gases a negative source of oxygen (sink) is connected. The gaseous mixture is treated as compressible, Newtonian, and viscous fluid. We assume that also the components of the mixture are compressible, Newtonian, viscous, noninteracting fluids (perfect gases). The i th component possesses individual velocity $\vec{v}_i(x, y, z, t)$, density $\rho_i(x, y, z, t)$, and pressure $p_i(x, y, z, t)$, whereas the mixture possesses total velocity $\vec{v}(x, y, z, t)$, density $\rho(x, y, z, t)$, pressure $p(x, y, z, t)$, and temperature $T(x, y, z, t)$.

2. 4. Equations of dynamics: Balances of total momentum of mixture, of total mass of mixture, of masses of components of mixture, of energy of mixture, and equation of state of mixture (averaged over constituents), as well as balances of numbers of vehicles and equations of states of vehicles [4-15].

2. 5. Model variables: Mixture's temperature $T(x, y, z, t)$, mixture's total velocity $\vec{v}(x, y, z, t)$, mixture's total density $\rho(x, y, z, t)$, mass concentration of i th constituent of mixture $c_i(x, y, z, t)$, mixture's total pressure $p(x, y, z, t)$.

2. 6. State variables: Vehicular densities $k_{l,vt}^s(x, t)$, vehicular velocities $\vec{w}_{l,vt}^s(x, t)$, pollutants' emissivities $e_{l,ct,vt}^s(x, t)$, where for $n_1 = n_L$ left lanes $s = 1$ and l is left lane's number, $l = 1, ..., n_L$, whereas for $n_2 = n_R$ right lanes $s = 2$ and l is right lane's number, $l = 1, ..., n_R$, vt is vehicular type's number, $vt = 1, ..., VT$, ct is emitted pollutant's number, $ct = 1, ..., CT$, VT is number of vehicular types, CT is number of types of emitted pollutants. We shall adopt above notation throughout entire article.

2. 7. Vector of control: $u_j = (g_j, C_j, F)$, $j = 1, ..., M, M = 2$, vectors of control on j th junction, where $g_j \in [g_{j,\min}, C_j - g_{j,orth}]$ are green's times, $C_j \in [C_{j,\min}, C_{j,\max}]$ are cycle's times, and F is offset's time between beginnings of cycles on two junctions. Vector of control:

$$u = (g_1, C_1, g_2, C_2, F). \tag{1}$$

The admissible control domain set for this 5-tuple in the simulation time period $T_S > 0$ is for $j = 1, 2$:

$$U^{adm} = \{(g_1, C_1, g_2, C_2, F) : C_j \in [C_{j,\min}, C_{j,\max}], g_j \in [g_{j,\min}, C_j - g_{j,orth}], F \in [F_{\min}, C_2 - \delta_F]\}, \tag{2}$$

where δ_F is discretization step in direction F of control space.

2. 8. Boundary conditions: We assume von Neumann's boundary conditions on canyon's walls and on road's surface (normal derivatives of functions are given), whereas we assume Dirichlet's boundary conditions on remaining three walls (functions are given). Boundary conditions follow from viscosity of gaseous mixture, since velocity of fluid vanishes on non-moving impervious surface.

2. 9. Initial conditions: We assume initial conditions with accordance with boundary conditions.

2. 10. Sources (emission processes):

2. 10. 1. Heat's source: $\sigma(x,y,z,t)$, the rate of change of the volume density of internal sources of energy connected with the production of heat by vehicular engines. We assume that the sources of energy are situated in n_s left or right lanes at $y = y_l^s$, at the level of the road $z = 0$:

$$\sigma(x,y,z,t) = [\sum_{s=1}^{2} \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} \sigma_{l,vt}^s(x,t) \cdot \chi_{D_l^s}(x,y,z)]/(bc), \tag{3}$$

where $\sigma_{l,vt}^s(x,t)$, are the rates of change of linear density of energy connected to heat produced by engines of vehicles of type *vt* on *l* th left or right lane, respectively.

$$D_l^s = \{(x,y,0) : (x,y,0) \in \Omega, y = y_l^s\},$$

are vehicular lanes, and

$$\chi_D(x,y,z) = 1, \text{for} (x,y,z) \in D, \quad \chi_D(x,y,z) = 0, \text{ for} (x,y,z) \notin D,$$

is characteristic function of set D.

2. 10. 2. Pollutants' masses' sources: $Set_{ct}(x,y,z,t)$, the rate of change of the volume density of internal sources (the emission rate) of ct th component of exhaust gases emitted by all vehicles in the canyon. We assume that the sources of exhaust gases are situated in n_s left or right lanes at $y = y_l^s$, at the level of the road $z = 0$:

$$Set_{ct}(x,y,z,t) = [\sum_{s=1}^{2} \sum_{l=1}^{n_s} \sum_{vt=1}^{VT} e_{l,ct,vt}^s(x,t) \cdot \chi_{D_l^s}(x,y,z)]/(bc). \tag{4}$$

$Set_{N_E}(x,y,z,t)$, the volume density of negative internal sources (the emission rate) of oxygen absorbed by all vehicles in the canyon. We assume that:

$$Set_{N_E}(x,y,z,t) = ONOX \cdot Set_{N_E-1}(x,y,z,t), \text{ where } ONOX = -0.5308.$$

2. 10. 3. Mixture's mass' source: $S(x,y,z,t)$, the rate of change of the volume density of internal sources of gaseous mixture consisting of exhaust gases and of oxygen.

The following relation holds: $S(x,y,z,t) = \sum_{ne=1}^{N_E} Set_{ne}(x,y,z,t).$ (5)

2. 11. Set of equations of dynamics of air mixture and of vehicles [4-16].

2. 11. 1. Balance of momentum of mixture - Navier Stokes equation.

$$\rho(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\nabla)\vec{v}) + S\vec{v} = -\nabla p + \eta\Delta\vec{v} + (\xi + \frac{\eta}{3})\nabla(\nabla \circ \vec{v}) + \vec{F},$$ (6)

where η is first viscosity coefficient ($\eta = 18.1 \cdot 10^{-6}[\frac{kg}{s \cdot m}]$ for air at temperature $T = 293[K]$), ξ is

second viscosity coefficient ($\xi = 15.6 \cdot 10^{-6}[\frac{kg}{s \cdot m}]$ for air at temperature $T = 293.16[K]$, [16]), $\vec{F} = \rho\vec{g}$ is

gravitational body force density, $\vec{g}$ is gravitational acceleration of Earth ($\vec{g} = (0,0,-9.81)[\frac{m}{s^2}]$).

2. 11. 2. Balance of mass of mixture - Equation of continuity.

$$\frac{\partial \rho}{\partial t} + div(\rho\vec{v}) = S.$$ (7)

2. 11. 3. Balances of masses of components of mixture- Diffusion equations.

2.11.3a. $\rho(\frac{\partial c_i}{\partial t} + \vec{v}\nabla c_i) = Set_i - c_i S + \sum_{m=1}^{N-1}(D_{im} - D_{iN})div[\rho\nabla(c_m + \frac{k_{T,m}}{T}\nabla T)], i = 1,...,N_E,$ (8)

2.11.3b. $\rho(\frac{\partial c_i}{\partial t} + \vec{v}\nabla c_i) = -c_i S + \sum_{m=1}^{N-1}(D_{im} - D_{iN})div[\rho\nabla(c_m + \frac{k_{T,m}}{T}\nabla T)], i = (N_E + 1),...,N,$ (9)

where $D_{im} = D_{mi}$ is the mutual diffusivity coefficient from component i to m, and D_{ii} is the autodiffusivity coefficient of component i, and $k_{T,i}$ is the thermodiffusion ratio of component i.The diffusivity coefficients and thermodiffusion ratios are constant and known [17]. Since the mixture is in motion we cannot neglect the convection term: $\vec{v}\nabla c_i$ We assume that the barodiffusion and gravitodiffusion coefficients are equal to zero.

2. 11. 4. Balance of energy of mixture.

$$\rho(\frac{\partial \varepsilon}{\partial t} + (\vec{v}\nabla)\varepsilon) =$$

$$= -(-\frac{1}{2}\vec{v}^2 + \varepsilon)S + \hat{\Pi}:\nabla\vec{v} + \sigma + div(-\vec{q}),$$ (10)

where ε is the mass density of intrinsic energy of the air mixture, $\hat{\Pi}$ is the stress tensor, $:$is contraction operation, and $\vec{q}$ is the flux of heat. We assume that [1]:

$$\varepsilon = \sum_{i=1}^{N} \varepsilon_i ,$$

$$\varepsilon_i = (1/m_i)\{c_i k_B T \exp(-\frac{m_i |\vec{g}| z}{k_B T}) \cdot [(-\frac{z}{c})(1 - \exp(-\frac{m_i |\vec{g}| c}{k_B T})) - (\exp(-\frac{m_i |\vec{g}| c}{k_B T}))]\} + \widetilde{\mu}_i c_i ,$$

$$\widetilde{\mu}_i = \frac{\mu_i}{m_i} ,$$

$$\mu_i = \{k_B T \{\ln[(c_i p) \cdot (k_B T)^{-c_{p,i}/k_B} \cdot (\frac{m_{air}}{m_i}) \cdot (\frac{2\pi h^2}{m_i})^{3/2}]\}\} + m_i |\vec{g}| z\} ,$$

$$\hat{\Pi}_{mk} = -p \cdot \delta_{mk} + \eta[(\frac{\partial v_m}{\partial x_k} + \frac{\partial v_k}{\partial x_m} - \frac{2}{3}\delta_{mk} div(\vec{v}))\frac{\partial v_k}{\partial x_m}] + \xi[(\delta_{mk} div(\vec{v}))^2], m,k = 1,...,3,$$

$$\hat{\Pi} : \nabla \vec{v} = \sum_{m=1}^{3} \sum_{k=1}^{3} \hat{\Pi}_{mk} \frac{\partial v_m}{\partial x_k} ,$$

$$\vec{q} = \sum_{i=1}^{N} [(\widetilde{\mu}_i + \frac{\beta_i T}{\alpha_{ii}})\vec{j}_i] + [(-\gamma) \cdot \nabla T] ,$$

$$\vec{j}_i = -\rho D_{ii}(\nabla c_i + \frac{k_{T,i}}{T}\nabla T) ,$$

$$\alpha_{ii} = [\rho D_{ii}]/[(\frac{\partial \widetilde{\mu}_i}{\partial c_i})_{(c_n)_{n=1..N,n\neq i},T,p}] ,$$

$$\beta_i = \rho D_{ii}(\frac{k_{T,i}}{T} - \frac{[(\frac{\partial \widetilde{\mu}_i}{\partial T})_{(c_n)_{n=1..N},p}]}{[(\frac{\partial \widetilde{\mu}_i}{\partial c_i})_{(c_n)_{n=1..N,n\neq i},T,p}]}) , \tag{11}$$

where $\varepsilon_i, i = 1,...,N,$ is the mass density of intrinsic energy of ith constituent of air the mixture, μ_i, is the chemical potential of ith constituent of air, m_i, is the molecular mass of the ith constituent, $c_{p,i}$ is the specific heat at constant pressure of ith constituent of air, $k_B = 1.3807 \cdot 10^{-23} [\frac{J}{K}]$ is Boltzmann's constant, $h = 6.62608 \cdot 10^{-34} [J \cdot s]$ is Planck's constant, γ is coefficient of thermal conductivity of air, and $\vec{j}$ is the flux of mass of the ith constituent. The above mentioned magnitudes are derived from Grand Canonical Ensemble with external gravitational field [1].

2. 11. 5. Equations of state - Constitutive equations-Clapeyron's equation and Dalton's law.

$$\textbf{2.11.5a}. \frac{p}{\rho} = \frac{R}{m_{air}} T , \tag{12}$$

$$\textbf{2.11.5b}. p_i = c_i \frac{m_i}{m_{air}} , \tag{13}$$

where $R = 8.3145 [\dfrac{J}{mole \cdot K}]$ is gas constant, $m_{air} = 28.966u (u = 1.66054 \cdot 10^{-27} [kg])$ is the molecular mass of air.

2. 11. 6. Balances of numbers of vehicles - Equations of continuity [4].

$$\frac{\partial k_{l,vt}^s}{\partial t} + div(k_{l,vt}^s \vec{w}_{l,vt}^s) = 0. \tag{14}$$

2. 11. 7. Equations of states of vehicles –Greenshields' equilibrium u-k model [5].

$$\vec{w}_{l,vt}^s(x,t) = (w_{l,vt,f}^s(1 - \frac{k_{l,vt}^s(x,t)}{k_{l,vt,jam}^s}),0,0). \tag{15}$$

where $w_{l,vt,f}^s$, are vehicular free flow speeds, and $k_{l,vt,jam}^s$, are vehicular jam densities [5].

2. 11. 8. Technical parameters.

The dependence of the emissivity on density and velocity of vehicles is taken in the form:

$$e_{l,ct,vt}^s(x,t) = k_{l,vt}^s(x,t)[(\frac{|\vec{w}_{l,vt}^s(x,t)| - v_{ct,vt,i_l}}{v_{ct,vt,i_l+1} - v_{ct,vt,i_l}}) \cdot (e_{ct,vt,i_l+1} - e_{ct,vt,i_l}) + e_{ct,vt,i_l}], \tag{16}$$

where $|\vec{w}_{l,vt}^s(x,t)| \in (v_{ct,vt,i_l}, v_{ct,vt,i_l+1})$, and v_{ct,vt,i_l}, are experimental velocities and e_{ct,vt,i_l}, are experimental emissions of ct th exhaust gas from single vehicle of vt th type at velocity v_{ct,vt,i_l}, measured in $[\dfrac{kg}{s \cdot veh}]$ [18-19].

Similarly, the dependence of the change of the linear density of energy on density and velocity of vehicles is taken in the form:

$$\sigma_{l,vt}^s(x,t) = q_{vt} k_{l,vt}^s(x,t)[(\frac{|\vec{w}_{l,vt}^s(x,t)| - v_{vt,i_l}}{v_{vt,i_l+1} - v_{vt,i_l}}) \cdot (\sigma_{l,vt,i_l+1} - \sigma_{l,vt,i_l}) + \sigma_{l,vt,i_l}], \tag{17}$$

where σ_{l,vt,i_l}, are experimental values of consumption of gasoline/diesel for vehicle of vt th type at velocity v_{vt,i_l}, measured in $[\dfrac{kg}{s \cdot veh}]$, q_{vt} is the emitted combustion energy per unit mass of gasoline/diesel $[\dfrac{J}{kg}]$ [18].

2. 11. 9. Vector of control.

$$u = (g_1, C_1, g_2, C_2, F) \in U^{adm}, \tag{18}$$

where U^{adm} is a set of admissible control variables (compare Eqs (1), (2)).

2. 12. Optimization problems.

Our control task is the minimization of the measures of the total travel time TTT [4], emissions E, and concentrations C of exhaust gases in the street canyon, therefore the six separate monocriterial optimization problems are formulated as follows:

F1. $\min\limits_{u \in U^{adm}} J_{TTT}(u)$, (19)

where $J_{TTT}(u) = \sum\limits_{s=1}^{2} \sum\limits_{l=1}^{n_s} \sum\limits_{vt=1}^{VT} \int\limits_{0}^{T_S} \int\limits_{0}^{a} k_{l,vt}^{s}(x,t)\,dt\,dx$. (20)

F2. $\min\limits_{u \in U^{adm}} J_{E}(u)$, (21)

where $J_{E}(u) = \sum\limits_{s=1}^{2} \sum\limits_{l=1}^{n_s} \sum\limits_{ct=1}^{CT} \sum\limits_{vt=1}^{VT} \int\limits_{0}^{T_S} \int\limits_{0}^{a} e_{l,ct,vt}^{s}(x,t)\,dt\,dx$. (22)

F3. $\min\limits_{u \in U^{adm}} J_{C}(u)$, (23)

where $J_{C}(u) = \rho_{STP} \sum\limits_{i=1}^{N_E-1} \int\limits_{0}^{T_S} \int\limits_{0}^{a} \int\limits_{0}^{b} \int\limits_{0}^{c} c_i(x,y,z,t)\,dt\,dx\,dy\,dz$. (24)

F4. $\min\limits_{u \in U^{adm}} J_{TTT,ext}(u)$, (25)

where $J_{TTT,ext}(u) = J_{TTT}(u) + \sum\limits_{s=1}^{2} \alpha_{TTT,ext}^{s} \left(a \sum\limits_{l=1}^{n_s} \sum\limits_{vt=1}^{VT} k_{l,vt,jam}^{s}\right)(C_s - g_s)$. (26)

F5. $\min\limits_{u \in U^{adm}} J_{E,ext}(u)$, (27)

where $J_{E,ext}(u) = J_{E}(u) + \sum\limits_{s=1}^{2} \left(a \sum\limits_{l=1}^{n_s} \sum\limits_{ct=1}^{CT} \sum\limits_{vt=1}^{VT} e_{l,ct,vt,jam}^{s}\right)(C_s - g_s)$. (28)

F6. $\min\limits_{u \in U^{adm}} J_{C,ext}(u)$, (29)

where $J_{C,ext}(u) = J_{C}(u) + \left(\rho_{STP} abc \sum\limits_{i=1}^{N_E-1} c_{i,STP}\right)\left(\sum\limits_{s=1}^{2} \alpha_{C,ext}^{s}(C_s - g_s)\right)$. (30)

Remark: The density of vehicles, the emissions and concentrations in **F1**, **F2**, **F3**, **F4**, **F5**, **F6,** are the solutions of equations with given boundary and initial conditions, and the sources. The functionals in **F1**, **F2**, **F3**, **F4**, **F5**, **F6,** depend on the vector of control u through the conditions given in the next Section 3, Eq. (31). The six separate monocriterial optimization problems **F1**, **F2**, **F3**, **F4**, **F5**, **F6,** are considered parallelly. J_{TTT} and $J_{TTT,ext}$ are measured in $[veh \cdot s]$, J_E and $J_{E,ext}$ are measured in $[kg]$, and J_C and $J_{C,ext}$ are measured in $[kg \cdot s]$. Moreover, $e_{l,ct,vt,jam}^{s}$, are the jam vehicular emissions, and $c_{i,STP}$ are the pollutants' concentrations at standard temperature and pressure STP, and ρ_{STP} is density of air at STP. In the last three optimization problems **F4, F5, F6,** we defined the additional cost functions ascribed to the nearest neighbour previous and next street canyon assuming that in these canyons there are full jams. Hence, by these additional terms we modelled the remaining canyons of the city.

3. Numerical examples: domination of passenger cars over trucks.

Now, we will solve numerically six separate monocriterial optimization problems **F1-F6.** We assumed the data from real street canyon Krasiński Avenue in Cracow [20]. We solve the set of equations of dynamics with given boundary

and initial conditions, and sources, by finite difference method using the C language programme written by the author. We solve this set in the cuboid Ω starting from initial conditions and we iterate it over the time period $[0, T_S]$ using the direct finite difference method taking into account the boundary conditions, and the sources, and initial conditions, at each time step. The functionals in **F1, F2, F3, F4, F5, F6,** are iterated with the same steps that the equations of dynamics are iterated. The temporal first derivative is approximated by first differential quotient using forward two-point first difference in direction of temporal coordinate, whereas the spatial first derivatives are approximated by first differential quotients using central three-point first differences in directions of spatial coordinates. In general, the numerical results are in very good agreement with measured data from [20]. We use the notation: $(C_{1TTT}, g_{1TTT}, C_{2TTT}, g_{2TTT}, F_{TTT})$ is vector of control for optimal total travel time **F1,** $(C_{1E}, g_{1E}, C_{2E}, g_{2E}, F_E)$ is vector of control for optimal emissions **F2,** $(C_{1C}, g_{1C}, C_{2C}, g_{2C}, F_C)$ is vector of control for optimal concentrations **F3.** $(C_{1TTT,ext}, g_{1TTT,ext}, C_{2TTT,ext}, g_{2TTT,ext}, F_{TTT,ext})$ is vector of control for optimal total travel time **F4,** $(C_{1E,ext}, g_{1E,ext}, C_{2E,ext}, g_{2E,ext}, F_{E,ext})$ is vector of control for optimal emissions **F5,** $(C_{1C,ext}, g_{1C,ext}, C_{2C,ext}, g_{2C,ext}, F_{C,ext})$ is vector of control for optimal concentrations **F6**. We assumed the following data from real street canyon Al. Krasiński avenue in Cracow [20]:

$$VT = 4, n_L = n_R = 2; \qquad CT = N_E - 1 = 3; \qquad a = 468[m], b = 44[m], c = 20[m], T = 60[s],$$

$\delta_x = 46.8[m], \delta_y = 8.80[m], \delta_z = 4.00[m], \delta_t = 4.8[s],$ the discretization steps in $x, y, z, t,$ directions,

$\delta_{C_1} = 15.0[s], \delta_{g_1} = 7.5[s], \delta_{C_2} = 15.0[s], \delta_{g_2} = 7.5[s], \delta_F = 7.5[s],$ the discretization steps in $C_1, g_1, C_2, g_2, F,$ directions, $C_{j,\min} = 30[s]; g_{j,\min} = 10[s], j = 1,2; F_{\min} = 0[s],$ the minimal values of control variables C_1, g_1, C_2, g_2, F and $C_{j,\max} = T, g_{j,\max} = C_j - g_{j,orth}, j = 1,2; F_{\max} = C_2 - \delta_F,$ the maximal value of F, $g_{j,orth} = 15[s], j = 1,2,$ are the green's lights' lengths on the orthogonal canyons to the one studied. The scaling parameters in **F1-F6** are set to unity: $\alpha_{TTT,ext}^1 = 1; \alpha_{TTT,ext}^2 = 1; \alpha_{E,ext}^1 = 1; \alpha_{E,ext}^2 = 1; \alpha_{C,ext}^1 = 1; \alpha_{C,ext}^2 = 1.$

According to [4] we assumed the vehicular boundary conditions in such a form: vehicular density is equal to saturation or arrival density respectively, if traffic signals are green and there is no queue or there is queue, respectively. If the traffic signals are red, then vehicular density is equal to jam density [1].

The existence of the queues at the entrances to the canyon at $x = 0,$ for the left lanes, and at $x = a,$ for the right lanes is determined by the values of the vehicular densities changing in the following way:

$$k_{l,vt}^1(-\delta_x, t) = k_{GREEN} \text{ for } t \in [0, g_1) \cup [C_1, C_1 + g_1) \cup ...,$$

$$k_{l,vt}^1(-\delta_x, t) = k_{RED} \text{ for } t \in [g_1, C_1) \cup [C_1 + g_1, 2C_1) \cup ...,$$

$$k_{r,vt}^2(a + \delta_x, t) = k_{GREEN} \text{ for } t \in [F, F + g_2) \cup [F + C_2, F + C_2 + g_2) \cup ...,$$

$$k_{r,vt}^2(a + \delta_x, t) = k_{RED} \text{ for } t \in [0, F) \cup [F + g_2, F + C_2 + g_2) \cup ...,$$

$$k_{GREEN} = 0.030[veh/m], k_{RED} = 0.006[veh/m], \tag{31}$$

where $k_{GREEN}, k_{RED},$ are green's, or red's vehicular densities, respectively [1].

From the numerical simulations we deduce that [1]:

I1. The optimal pollutants' emissions **F2, F5,** are the lowest if there are no vehicles on left (leeward) and right (windward) lanes; then, they are greater if the vehicles are only on left or right lanes (and they are equal); finally, they are the greatest if there are vehicles on both lanes. The optimal pollutants' concentrations **F3, F6,** are the lowest if there are no vehicles on left (leeward) and right (windward) lanes; then, they are greater if the vehicles are only on left lanes; next, they are again greater if there are vehicles on both lanes; finally, they are the greatest if there are vehicles only on right lanes.

I2. In the cases **F1, F2, F3,** if there are no vehicles on left and right lanes, then optimal total travel time **F1** and optimal pollutants' emissions **F2** are equal to zero, whereas optimal concentrations **F3** are not equal to zero, since the pollutants are dispersed in the air even in the absence of vehicles (background pollutants' concentrations). In the **F4, F5, F6,** cases all values are nonzero.

I3. The optimal 5-tuples in the **F1, F2, F3,** cases are always different (no triple degeneration) with only one exception for the absence of vehicles (triple degeneration of 5-tuples). In some cases, there is double degeneration between 5-tuples for **F2** and **F3.**

I4. For the optimal 5-tuple for total travel time **F1,** $C_{1TTT}, C_{2TTT}, g_{1TTT}, g_{2TTT}$, tend to be minimal (the minimal capacity on both left and right lanes), only in the absence of vehicles on both lanes. For the right lanes, the cycle's time is always maximal. For the left lanes, the cycle's time is always maximal, and green's time is equal to, or is essentially longer, than cycle's time for right lanes. This is a result of coordination trade-offs between the traffic demands on the left and right lanes.

I5. If the optimal 5-tuples for **F2, F3,** cases are different from the 5-tuple for total travel time **F1,** then they are asymmetrical: $(C_{1E} \neq C_{2E})$, or $(g_{1E} \neq g_{2E})$, for **F2;** similarly $(C_{1C} \neq C_{2C})$, or $(g_{1C} \neq g_{2C})$, for **F3.** The optimal offset's times F_{TTT}, F_E, F_C, are nontrivial parameters.

I6. The optimal 5-tuples in the **F4, F5, F6,** cases are always different (no triple degeneration) with only one exception for the absence of vehicles (triple degeneration of 5-tuples). In some cases, there is double degeneration between 5-tuples for **F5** and **F6.**

I7. For the left lanes the cycle's times and green's times are equal to, or are essentially longer, than for right lanes for **F4** with one exception for the absence of vehicles, and the optimal offset's time is a nontrivial parameter. The green's times for left lanes are always great. This is a result of coordination trade-offs between the traffic demands on the left and right lanes.

I8. For **F5, F6,** the optimal offset's times are nontrivial parameters. The green's times and cycle's times for right lanes are sometimes minimal. This is a result of coordination trade-offs between the traffic demands on the left and right lanes.

I9. If the optimal 5-tuples for **F5, F6,** cases are different from the 5-tuple of **F4,** then the optimal offset's times for **F5, F6,** are nontrivial parameters.

I10. The 5-tuples for **F1** and **F4** are sometimes identical.

I11. The 5-tuples for **F2** and **F5,** as well as **F3** and **F6,** are nonidentical with exception of absence of vehicles.

I12. The optimal total travel time values for **F4** case are always greater than for **F1** case.

I13. The optimal emission values for **F5** case are always greater than for **F2** case.

I14. The optimal pollutants' concentrations' values for **F6** case are always greater than for **F3** case.

I15. The optimal values for **F1, F4,** cases, for **F2, F5,** cases, and for **F3, F6,** cases, decrease with "uniformization" of vehicles, when we pass from nonuniform vehicles to uniform ones. It is a result of decrement of the number of vehicles moving in the canyon. For uniform vehicles the values of jam, saturation, threshold, green's, and red's densities take on minima.

I16. The long vehicular queues decrease total travel times **F1, F4,** and they increase both optimal emissions **F2, F5,** and concentrations of pollutants **F3, F6.** The decrement of total travel times **F1, F4,** with long vehicular queues is result of clustering of vehicles.

We define ratios of dimensionless the worst solutions of problems **F1, F2, F3, F4, F5, and F6,** to corresponding optimal solutions: $r_{TTT}, r_E, r_C, r_{TTT,ext2}, r_{E,ext}, r_{C,ext}$, respectively. The worst solutions are the antioptimal solutions, *e. g.,* instead of minima we take maxima in problems **F1, F2, F3, F4, F5, F6.**

I16. Generally, we gain a lot in the case of total travel time for single canyon r_{TTT} **F1;** as well as in pollutants' emissions for both single canyon r_E **F2,** and for single canyon with two substitute nearest neighbour canyons $r_{E,ext}$ **F5.** It is even possible to obtain ratios greater than two. It means; that, if we *optimally* control street canyon, then, we can reduce more than one and half the total travel time, and more than two emissions. We know it, because we calculated the worst control scenarios. However, it is not the same for the remaining ratios. The ratios for pollutants' concentrations for single canyon r_C **F3;** as well as both ratios for total travel time $r_{TTT,ext2}$ **F4,** and pollutants' concentrations $r_{C,ext}$ **F6,** for single canyon with two substitute nearest neighbour canyons are very close to one. This fact means, that we gain almost nothing by optimal control of street canyon, and that the functionals **F3, F4, F6,** are very flat (almost constant) in the range of calculations and of parameters. It could be understood for pollutants' concentration cases **F3, F6,** since the masses of pollutants and of remaining gases in canyons are huge, and it is not

possible to remove them from canyon. In the case of total travel time **F4,** probably, we must take into consideration influence of other than two canyons of urban network.

I17. The ratios r_{TTT} **F1** for total travel times are not defined if there are no vehicles on left (leeward) and right (windward) lanes; then, they are equal for three cases: if the vehicles are only on left lanes, or only right lanes, or on both lanes. They increase with increment of queues and they are insensible for "uniformization" of vehicular parameters. Also, the ratios r_E **F2** for pollutants' emissions are not defined if there are no vehicles on left (leeward) and right (windward) lanes; then, they are equal for three cases: if the vehicles are only on left or only right lanes, or on both lanes. They increase with increment of queues and they decrease with "uniformization" of vehicular parameters. The ratios r_C **F3** for pollutants' concentrations are equal to unity for all cases.

I18. The ratios $r_{TTT,ext}$ **F4,** for total travel times for canyon with two substitute nearest neighbour canyons are always equal to unity. The ratios $r_{E,ext}$ **F5,** for pollutants' emissions for canyon with two substitute nearest neighbour canyons are the highest if there are no vehicles on left (leeward) and right (windward) lanes; then, they are lower and equal for two cases: if the vehicles are only on left or only right lanes; finally, they are the lowest, if there are vehicles on both left and right lanes. They decrease both with increment of queues and with "uniformization" of vehicular parameters. They are almost always higher than emission ratios for single canyon r_E **F2**. This effect is interesting, because it shows a countertendency to emission ratios for single canyon r_E **F2** (see **I17**) and to optimal emissions **F2, F5** (compare **I1**). Thus, the six functionals **F1, F2, F3, F4, F5, F6,** are nontrivially chosen, and they show interesting features. Also, the inclusion of substitute canyons shows here its importance, *e. g.,* that r_E **F2** and $r_{E,ext}$ **F5** contrarily feel the influence of surrounding urban network. The ratios $r_{C,ext}$ **F6** for pollutants' concentrations for canyon with two substitute nearest neighbour canyons are the highest if there are no vehicles on left (leeward) and right (windward) lanes; then, they are lower if the vehicles are only on left lanes; next, they are lower, if the vehicles are on both left and right lanes; finally, they are the lowest, if the vehicles are only on right lanes. They increase both with increment of queues and with "uniformization" of vehicular parameters with one exception. They are always higher that concentration ratios for single canyon r_C **F3**. The ratios $r_{C,ext}$ **F6** are close to unity.

I19. The ratios in the **F1, F2, F3,** cases are always different (neither double nor triple degeneration) with only one exception for the absence of vehicles (where two ratios are not defined).

I20. The ratios in the **F4, F5, F6,** cases are always different (neither double nor triple degeneration). There is no double degeneration between ratios r_{TTT}, $r_{TTT,ext}$, between r_E, $r_{E,ext}$, and between r_C, $r_{C,ext}$.

4. Conclusions.

The article is concerned with formulation and solution of optimal road traffic control problem with hydrodynamical model of the street canyon. The article provided nontrivial dynamical spatially three-dimensional, temporally dependent, field model of vehicular movement, of emissions, and of dynamics of pollutants in the street canyons. The several control optimization problems were formulated and solved: we calculated minimal total travel time, global emissions, and global concentrations of pollutant, in single canyon, and in canyon with two nearest neighbour substitute canyons. The numerical examples of different traffic and meteorological scenarios were provided and conclusions were inferred.

In general, some vehicular traffic and hydrodynamical parameters influence the solutions of optimization problems **F1, F2, F3, F4, F5, F6,** but not all of them:

R1. The direction of velocity of air mixture is important. The optimal pollutants' concentrations for both single canyon and canyon with its two substitute nearest neighbour canyons, **F3, F6,** are the lowest if the velocity components of the boundary and initial value problems are equal to zero; further, they are the grater if velocity has only nonzero vertical component; then, they are greater if velocity has only nonzero x-component; further, they are again greater if velocity has two nonzero y- and z-components; next, they are again greater if velocity has three nonzero x-, y-, z-components; finally, they are the greatest if the velocity has only nonzero y-component. From numerical simulations, we infer that in some case, the optimal green's time $g_{2,C}$ is different from other cases. It means that velocity of mixture influences optimal 5-tuples of control.

R2. The optimal values for **F1, F4,** cases, for **F2, F5,** cases, and for **F3, F6,** cases, decrease with "uniformization" of vehicles, when we pass from nonuniform vehicles to uniform ones. It is a result of decrement of the number of vehicles moving in the canyon. For uniform vehicles the values of maximum free flow speed, jam, saturation, threshold, green's, and red's densities take on minima.

R3. The long vehicular queues decrease total travel times **F1, F4,** and they increase both optimal emissions **F2, F5,** and concentrations of pollutants **F3, F6.** The decrement of total travel times **F1, F4,** with long vehicular queues is result of clustering of vehicles.

R4. The constant of temperature scale TH does not differentiate the values of optimal concentrations of pollutants **F3, F6,** in the temperature range near standard temperature and pressure STP conditions. However, it diminishes them even hundredfold for very high temperatures.

R5. The functional form of initial and boundary conditions affects the optima. If they are constant then optimal concentrations **F3, F6,** are twice higher than in the case when they are changing exponentially in space in three dimensions.

R6. The presence of vehicles on both left and right lanes is important. The optimal total travel times and emissions are halved in absence of vehicles only on left or right lanes with respect to situation when they circulate on both left and right lanes.

R7. The values of saturation, arrival, or jam vehicular density, and of vehicular free flow velocities also affect the optima **F1, F2, F3, F4, F5, F6.**

R8. The assumption of energy conservation equation, of thermodiffusion effect, of chemical potential and of Grand Canonical ensemble, as well as of influence of gravity on intrinsic energy and on chemical potential, drastically changes the optimal concentrations **F3, F6,** towards measured ones [1-3].

R9. The value of time of simulation and of discretization in time affects much the optima. The values of optimal solutions **F1, F2, F3, F4, F5, F6,** increase from tenfold to hundredfold. Also the optimal 5-tuples for **F1, F2, F3, F4, F5, F6,** change their values. It is result of cumulative effect of length of period of simulation T_S on integral functionals **F1, F2, F3, F4, F5, F6.**

The proecological traffic control problem and advanced model of the street canyon have been developed in the article. It was found that the proposed model represents the relevant features of the very complex air pollution phenomena. The control model of the street canyon may be in a simple way extended by three-dimensional representations of vehicles, multilevel streets and junctions, as well as canyons with nonhomogeneous walls. Furthermore, it can be remodelled to artery models or to urban traffic subnetwork models.

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